Aether and Matter

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CHAPTER X:

GENERAL PROBLEM OF MOVING MATTER TREATED IN RELATION TO THE INDIVIDUAL MOLECULES

Formulation of the Problem

102. We shall now consider the material system as consisting of free aether pervaded by a system of electrons which are to be treated individually, some of them free or isolated, but the great majority of them grouped into material molecules: and we shall attempt to compare the relative motions of these electrons when they form, or belong to, a material system devoid of translatory motion through the aether, with what it would be when a translatory velocity is superposed, say for shortness a velocity v parallel to the axis of x. The medium in which the activity occurs is for our present purpose the free aether itself, whose dynamical equations have been definitely ascertained in quite independent ways from consideration of both the optical side and the electrodynamic side of its activity: so that there will be nothing hypothetical in our analysis on that score. An electron e will occur in this analysis as a singular point in the aether, on approaching which the elastic strain constituting the aethereal displacement $(f,\ g,\ h)$ increases indefinitely, according to the type

$$-e/4\pi\ .\ (d/dx,\ d/dy,\ d/dz)r^{-1}\ :$$

it is in fact analogous to what is called a simple pole in the two-dimensional representation that is employed in the theory of a function of a complex variable. It is assumed that this singularity represents a definite structure, forming a nucleus of strain in the aether, which is capable of transference across that medium independently of motion of the aether itself: the portion of the surrounding aethereal strain, of which the displacement-vector $(f,\ g,\ h)$ is the expression, which is associated with the electron and is carried along with the electron in its motion, being as above $-e/4\pi\ .\ (d/dx,\ d/dy,\ d/dz)r^{-1}$. It is to be noticed that the energy of this part of the displacement is closely concentrated around the nucleus of the electron, and not widely diffused as might at first sight appear. The aethereal displacement satisfies the stream-condition

$$df/dx + dg/dy + dh/dz = 0,$$

except where there are electrons in the effective element of volume: these are analogous to the so-called sources and sinks in the abstract theory of liquid flow, so that when electrons are present the integral of the normal component of the aethereal displacement over the boundary of any region, instead of being null, is equal to the quantity Σe of electrons existing in the region. The other vector which is associated with the aether, namely the magnetic induction $(a,\ b,\ c)$, also possesses the stream property; but singular points in its distribution, of the nature of simple poles, do not exist. The motion of an electron involves however a singularity in $(a,\ b,\ c)$, of a rotational type, with its nucleus at the moving electron;[1] and the time-average of this singularity for a very rapid minute steady orbital motion of an electron is analytically equivalent, at distances considerable compared with the dimensions of the orbit, to a magnetic doublet analogous to a source and associated equal sink. Finally, the various parts of the aether are supposed to be sensibly at rest, so that for example the time-rate of change of the strain of any element of the aether is represented by differentiation with respect to the time without any additional terms to represent the change due to the element of aether being carried on in the meantime to a new position; in this respect the equations of the aether are much simpler than those of the dynamics of fluid motion, being in fact linear. The aether is stagnant on this theory, while the molecules constituting the Earth and all other material bodies flit through it without producing any finite flow in it; hence the law of the astronomical aberration of light is rigorously maintained, and the Doppler change of wave-length of radiation from a moving source holds good; but it will appear that all purely terrestrial optical phenomena are unaffected by the Earth's motion.

103. Subject to this general explanation, the analytical equations which express the dynamics of the field of free aether, existing between and around the nuclei of the electrons, are

$$4\pi\frac{d}{dt}(f,\ g,\ h) = curl\ (a,\ b,\ c)$$

$$-\frac{d}{dt}(a,\ b,\ c) = 4\pi c^2 curl\ (f,\ g,\ h),$$

in which the symbol $curl\ (a,\ b,\ c)$ represents, after Maxwell, the vector

$$\left(\frac{dc}{dy} - \frac{db}{dz},\ \frac{da}{dz} - \frac{dc}{dx},\ \frac{db}{dx} - \frac{da}{dy}\right),$$

and in which c is the single physical constant of the aether, being the velocity of propagation of elastic disturbances through it. These are the analytical equations derived by Maxwell in his mathematical development of Faraday's views as to an electric medium: and they are the same as the equations arrived at by MacCullagh a quarter of a century earlier in his formulation of the dynamics of optical media. It may fairly be claimed that the theoretical investigations of Maxwell, in combination with the experimental verifications of Hertz and his successors in that field, have imparted to this analytical formulation of the dynamical relations of free aether an exactness and precision which is not surpassed in any other department of physics, even in the theory of gravitation.

Where a more speculative element enters is in the construction of a kinematic scheme of representation of the aether-strain, such as will allow of the unification of the various assumptions here enumerated. It is desirable for the sake of further insight, and even necessary for various applications, to have concrete notions of the physical nature of the vectors $(f,\ g,\ h)$ and $(a,\ b,\ c)$ which specify aethereal disturbances, in the form of representations such as will implicitly and intuitively involve the analytical relations between them, and will also involve the conditions and restrictions to which each is subject, including therein the permanence and characteristic properties of an electron and its free mobility through the aether.[2]

104. But for the mere analytical development of the aether-scheme as above formulated, a concrete physical representation of the constitution of the aether is not required: the abstract relations and conditions above given form a sufficient basis. In point of fact these analytical relations are theoretically of an ideal simplicity for this purpose: for they give explicitly the time-rates of change of the vectors of the problem at each instant, so that from a knowledge of the state of the system at any time t the state at the time $t + \delta t$ can be immediately expressed, and so by successive steps, or by the use of Taylor's differential expansion-theorem, its state at any further time can theoretically be derived. The point that requires careful attention is as to whether the solution of these equations in terms of a given initial state of the system determines the motions of the electrons or strain-nuclei through the medium, as well as the changes of strain in the medium itself: and it will appear on consideration that under suitable hypotheses this is so. For the given initial state will involve given motions of the electrons, that is the initial value of $(a,\ b,\ c)$ will involve rotational singularities at the electrons around their directions of motion, just such as in the element of time δt will shift the electrons themselves into their new positions:[3] and so on step by step continually. This however presupposes that the nucleus of the electron is quite labile as regards displacement through the aether, in other words that its movement is not influenced by any inertia or forces except such as are the expression of its relation to the aether: we in fact assume the *completeness* of the aethereal scheme of relations as above given. Any difficulty that may be felt on account of the infinite values of the vectors at the nucleus itself may be removed, in the manner customary in analytical discussions on attractions, by considering the nucleus to consist of a volume distribution of electricity of finite but very great density, distributed through a very small space instead of being absolutely concentrated in a point: then the quantities will not become infinite. Of the detailed structure of electrons nothing is assumed: so long as the actual dimensions of their nuclei are extremely small in comparison with the distances between them, it will suffice for the theory to consider them as points, just as for example in the general gravitational theory

of the Solar System it suffices to consider the planets as attracting points. This method is incomplete only as regards those portions of the energy and other quantities that are associated with the mutual actions of the parts of the electron itself, and are thus molecularly constitutive.

105. It is to be observed that on the view here being developed, in which atoms of matter are constituted of aggregations of electrons, the only actions between atoms are what may be described as electric forces. The electric character of the forces of chemical affinity was an accepted part of the chemical views of Davy, Berzelius, and Faraday; and more recent discussions, while clearing away crude conceptions, have invariably tended to the strengthening of that hypothesis. The mode in which the ordinary forces of cohesion could be included in such a view is still quite undeveloped. Difficulties of this kind have however not been felt to be fundamental in the vortex-atom illustration of the constitution of matter, which has exercised much fascination over high authorities on molecular physics: yet in the concrete realization of Maxwell's theory of the aether above referred to, the atom of matter possesses all the dynamical properties of a vortex ring in a frictionless fluid, so that everything that can be done in the domain of vortex-ring illustration is implicitly attached to the present scheme. The fact that virtually nothing has been achieved in the department of forces of cohesion is not a valid objection to the development of a theory of the present kind. For the aim of theoretical physics is not a complete and summary conquest of the *modus operandi* of natural phenomena: that would be hopelessly unattainable if only for the reason that the mental apparatus with which we conduct the search is itself in one of its aspects a part of the scheme of Nature which it attempts to unravel. But the very fact that this is so is evidence of a correlation between the process of thought and the processes of external phenomena, and is an incitement to push on further and bring out into still clearer and more direct view their inter-connexions. When we have mentally reduced to their simple elements the correlations of a large domain of physical phenomena, an objection does not lie because we do not know the way to push the same principles to the explanation of other phenomena to which they should presumably apply, but which are mainly beyond the reach of our direct examination.

The natural conclusion would rather be that a scheme, which has been successful in the simple and large-scale physical phenomena that we can explore in detail, must also have its place, with proper modifications or additions on account of the difference of scale, in the more minute features of the material world as to which direct knowledge in detail is not available. And in any case, whatever view may be held as to the necessity of the whole complex of chemical reaction being explicable in detail by an efficient physical scheme, a limit is imposed when vital activity is approached: any complete analysis of the conditions of the latter, when merely superficial sequences of phenomena are excluded, must remain outside the limits of our reasoning faculties. The object of scientific explanation is in fact to coordinate mentally, but not to exhaust, the interlaced maze of natural phenomena: a theory which gives an adequate correlation of a portion of this field maintains its place until it is proved to be in definite contradiction, not removable by suitable modification, with another portion of it.

Application to moving Material Media: approximation up to first order

106. We now recall the equations of the free aether, with a view to changing from axes (x, y, z) at rest in the aether to axes (x', y', z') moving with translatory velocity v parallel to the axis of x; so as thereby to be in a position to examine how phenomena are altered when the observer and his apparatus are in uniform motion through the stationary aether. These equations are

$$4\pi\frac{df}{dt} = \frac{dc}{dy} - \frac{db}{dz} \qquad -(4\pi c^2)^{-1}\frac{da}{dt} = \frac{dh}{dy} - \frac{dg}{dz}$$

$$4\pi\frac{dg}{dt} = \frac{da}{dz} - \frac{dc}{dx} \qquad -(4\pi c^2)^{-1}\frac{db}{dt} = \frac{df}{dz} - \frac{dh}{dx}$$

$$4\pi\frac{dh}{dt} = \frac{db}{dx} - \frac{da}{dy} \qquad -(4\pi c^2)^{-1}\frac{dh}{dt} = \frac{dg}{dx} - \frac{df}{dy}.$$

When they are referred to the axes *(x', y', z')* in uniform motion, so that $(x',\ y',\ z') = (x - vt,\ y,\ z),\ t' = t,$ then $d/dx,\ d/dy,\ d/dz$ become $d/dx',\ d/dy',\ d/dz'$, but *d/dt* becomes $d/dt' - vd/dx'$: thus

$$4\pi\tfrac{df}{dt'} = \tfrac{dc'}{dy'} - \tfrac{db'}{dz'} \qquad -(4\pi c^2)^{-1}\tfrac{da}{dt'} = \tfrac{dh'}{dy'} - \tfrac{dg'}{dz'}$$

$$4\pi\tfrac{dg}{dt'} = \tfrac{da'}{dz'} - \tfrac{dc'}{dx'} \qquad -(4\pi c^2)^{-1}\tfrac{db}{dt'} = \tfrac{df'}{dz'} - \tfrac{dh'}{dx'}$$

$$4\pi\tfrac{dh}{dt'} = \tfrac{db'}{dx'} - \tfrac{da'}{dy'} \qquad -(4\pi c^2)^{-1}\tfrac{dh}{dt'} = \tfrac{dg'}{dx'} - \tfrac{df'}{dy'}.$$

where

$$(a',\ b',\ c') = (a,\ b + 4\pi vh,\ c - 4\pi vg)$$

$$(f',\ g',\ h') = \left(f,\ g - \frac{v}{4\pi c^2}c,\ h + \frac{v}{4\pi c^2}b\right).$$

We can complete the elimination of (*f, g, h*) and (*a, b, c*) so that only the vectors denoted by accented symbols shall remain, by substituting from these latter formulae: thus

$$g = g' + \frac{v}{4\pi c^2}(c' + 4\pi vg),$$

so that

$$\epsilon^{-1}g = g' + \frac{v}{4\pi c^2}c',$$

where ε is equal to $\left(1 - v^2/c^2\right)^{-1}$, and exceeds unity;

and

$$b = b' - 4\pi v\left(h' - \frac{v}{4\pi c^2}b\right)$$

so that

$$\epsilon^{-1}b = b' - 4\pi vh';$$

giving the general relations

$$\epsilon^{-1}(a,\ b,\ c) = \left(\epsilon^{-1}a',\ b' - 4\pi vh',\ c' + 4\pi vg'\right)$$

$$\epsilon^{-1}(f,\ g,\ h) = \left(\epsilon^{-1}f',\ g' + \frac{v}{4\pi c^2}c',\ h' - \frac{v}{4\pi c^2}b'\right).$$

Hence

$$4\pi\frac{df'}{dt'} = \frac{dc'}{dy'} - \frac{db'}{dz'}$$

$$4\pi\epsilon\frac{dg'}{dt'} = \frac{da'}{dz'} - \left(\frac{d}{dx'} + \frac{v}{c^2}\epsilon\frac{d}{dt'}\right)c'$$

$$4\pi\epsilon\frac{dh'}{dt'} = \left(\frac{d}{dx'} + \frac{v}{c^2}\epsilon\frac{d}{dt'}\right)b' - \frac{da'}{dy'}$$

$$-\left(4\pi c^2\right)^{-1}\frac{da'}{dt'} = \frac{dh'}{dy'} - \frac{dg'}{dz'}$$

$$-\left(4\pi c^2\right)^{-1}\epsilon\frac{db'}{dt'} = \frac{df'}{dz'} - \left(\frac{d}{dx'} + \frac{v}{c^2}\epsilon\frac{d}{dt'}\right)h'$$

$$-\left(4\pi c^2\right)^{-1}\epsilon\frac{dc'}{dt'} = \left(\frac{d}{dx'} + \frac{v}{c^2}\epsilon\frac{d}{dt'}\right)g' - \frac{df'}{dy'}.$$

Now change the time-variable from t' to t'', equal to $t' - \frac{v}{c^2}\epsilon x'$; this will involve that $\frac{d}{dx'} + \frac{v}{c^2}\epsilon\frac{d}{dt'}$ is replaced by $\frac{d}{dx'}$, while the other differential operators remain unmodified; thus the scheme of equations reverts to the same type as when it was referred to axes at rest, except as regards the factors ε on the left-hand sides.

107. It is to be observed that this factor ε only differs from unity by $(v/c)^2$, which is of the second order of small quantities; hence we have the following correspondence when that order is neglected. Consider any aethereal system, and let the sequence of its spontaneous changes referred to axes (x', y', z') moving uniformly through the aether with velocity $(v, 0, 0)$ be represented by values of the vectors (f, g, h) and (a, b, c) expressed as functions of x', y', z' and t', the latter being the time measured in the ordinary manner: then there exists a correlated aethereal system whose sequence of spontaneous changes referred to axes (x', y', z') at rest are such that its electric and magnetic vectors (f', g', h') and (a', b', c') are functions of the variables x', y', z' and a time-variable t'', equal to $t' - \frac{v}{c^2}x'$, which are the same as represent the quantities

$$\left(f,\ g - \frac{v}{4\pi c^2}c,\ h + \frac{v}{4\pi c^2}b\right)$$

and

$$(a,\ b + 4\pi vh,\ c - 4\pi vg)$$

belonging to the related moving system when expressed as functions of the variables x', y', z' and t'.

Conversely, taking any aethereal system at rest in the aether, let the sequence of its changes be represented by *(f', g', h')* and *(a', b', c')* expressed as functions of the coordinates *(x, y, z)* and of the time *t'*. In these functions change *t'* into $t - \frac{v}{c^2}x$: then the resulting expressions are the values of

$$\left(f,\ g - \frac{v}{4\pi c^2}c,\ h + \frac{v}{4\pi c^2}b\right),$$

and

$$(a,\ b + 4\pi vh,\ c - 4\pi vg),$$

for a system in uniform motion through the aether, referred to axes (*x, y, z*) moving along with it, and to the time *t*. In comparing the states of the two systems, we have to the first order

$$\begin{array}{ll} \frac{df}{dx} & \text{equal to } \frac{df'}{dx} - \frac{v}{c'}\frac{df'}{dt} \\ \frac{d}{dy}\left(g - \frac{v}{4\pi c^2}c\right) & \quad ,, \quad \frac{dg'}{dy} \\ \frac{d}{dz}\left(h + \frac{v}{4\pi c^2}b\right) & \quad ,, \quad \frac{dh'}{dz}; \end{array}$$

hence bearing in mind that for the system at rest

$$\frac{dc'}{dy} - \frac{db'}{dz} = 4\pi\frac{df'}{dt'},$$

or, what is the same,

$$\frac{dc'}{dy} - \frac{db'}{dz} = 4\pi\left(\frac{df'}{dt} - v\frac{df'}{dx}\right),$$

we have, to the first order,

$$\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = \frac{df'}{dx} + \frac{dg'}{dy} + \frac{dh'}{dz}.$$

Thus the electrons in the two systems here compared, being situated at the singular points at which the concentration of the electric displacement ceases to vanish, occupy corresponding positions. Again, these electrons are of equal strengths: for, very near an electron, fixed or moving, the values of (*f, g, h*) and (*a, b, c*) are practically those due to it, the part due to the remainder of the system being negligible in comparison: also in this correspondence the relation between (*f, g, h*) and the accented variables is, by § 106

$$\epsilon^{-1}(f,\ g,\ h) = \left(\epsilon^{-1}f',\ g' + \frac{v}{4\pi c^2}c',\ h' - \frac{v}{4\pi c^2}b'\right);$$

hence, since for the single electron at rest *(a', b', c')* is null, we have, very close to the correlative electron in the moving system, *(f, g, h)* equal to *(f', εg', εh')*, where *ε*, being $\left(1 - v^2/c^2\right)^{-1}$, differs from unity by the second order of small quantities. Thus neglecting the second order, *(f, g, h)* is equal to *(f', g', h')* for corresponding points very close to electrons; and, as the amount of electricity inside any boundary is equal

to the integral of the normal component of the aethereal displacement taken over the boundary, it follows by taking a very contracted boundary that the strengths of the corresponding electrons in the two systems are the same, to this order of approximation.

108. It is to be observed that the above analytical transformation of the equations applies to any isotropic dielectric medium as well as to free aether: we have only to alter *c* into the velocity of radiation in that medium, and all will be as above. The transformation will thus be different for different media. But we are arrested if we attempt to proceed to compare a moving material system, treated as continuous, with the same system at rest; for the motion of the polarized dielectric matter has altered the mathematical type of the electric current. It is thus of no avail to try to effect in this way a direct general transformation of equations of a material medium in which dielectric and conductive coefficients occur.

109. The correspondence here established between a system referred to fixed axes and a system referred to moving axes will assume a very simple aspect when the former system is a steady one, so that the variables are independent of the time. Then the distribution of electrons in the second system will be at each instant precisely the same as that in the first, while the second system accompanies its axes of reference in their uniform motion through the aether. In other words, given any system of electrified bodies at rest, in equilibrium under their mutual electric influences and imposed constraints, there will be a precisely identical system in equilibrium under the same constraints, and in uniform translatory motion through the aether. That is, uniform translatory motion through the aether does not produce any alteration in electric distributions as far as the first order of the ratio of the velocity of the system to the velocity of radiation is concerned. Various cases of this general proposition will be verified subsequently in connexion with special investigations.

Moreover this result is independent of any theory as to the nature of the forces between material molecules: the structure of the matter being assumed unaltered to the first order by motion through the aether, so too must be all electric distributions. What has been proved comes to this, that if any configuration of ionic charges is the natural one in a material system at rest, the maintenance of the same configuration as regards the system in uniform motion will not require the aid of any new forces. The electron *taken by itself* must be on any conceivable theory a simple singularity of the aether whose movements when it is free, and interactions with other electrons if it can be constrained by matter, are traceable through the differential equations of the surrounding free aether alone: and a correlation has been established between these equations for the two cases above compared. It is however to be observed (cf. § 99) that though the fixed and the moving system of electrons of this correlation are at corresponding instants identical, yet the electric and magnetic displacements belonging to them differ by terms of the first order.

CHAPTER XI:
MOVING MATERIAL SYSTEM:

Approximation carried to the second order

110. THE results above obtained have been derived from the correlation developed in § 106, up to the first order of the small quantity *v/c*, between the equations for aethereal vectors here represented by *(f', g', h')* and *(a', b', c')* referred to the axes *(x', y', z')* at rest in the aether and a time *t''*, and those for related aethereal vectors represented by (*f, g, h*) and (*a, b, c*) referred to axes *(x', y', z')* in uniform translatory motion and a time *t'*. But we can proceed further, and by aid of a more complete transformation institute a correspondence which will be correct to the second order. Writing as before *t''* for $t' - \frac{v}{c^2}\epsilon x'$, the exact equations for (*f, g, h*) and (*a, b, c*) referred to the moving axes *(x', y', z')* and time *t'* are, as above shown, equivalent to

$$4\pi \frac{df'}{dt''} = \frac{dc'}{dy'} - \frac{db'}{dz'} \qquad -\left(4\pi c^2\right)^{-1} \frac{da'}{dt''} = \frac{dh'}{dy'} - \frac{dg'}{dz'}$$

$$4\pi\epsilon \frac{dg'}{dt''} = \frac{da'}{dz'} - \frac{dc'}{dx'} \qquad -\left(4\pi c^2\right)^{-1} \epsilon \frac{db'}{dt''} = \frac{df'}{dz'} - \frac{dh'}{dx'}$$

$$4\pi\epsilon \frac{dh'}{dt''} = \frac{db'}{dz'} - \frac{da'}{dy'} \qquad -\left(4\pi c^2\right)^{-1} \epsilon \frac{dc'}{dt''} = \frac{dg'}{dx'} - \frac{df'}{dy'}.$$

Now write

$\left(x_1,\ y_1,\ z_1\right)$ for $\left(\epsilon^{\frac{1}{2}} x',\ y',\ z'\right)$

$\left(a_1,\ b_1,\ c_1\right)$ for $\left(\epsilon^{-\frac{1}{2}} a',\ b',\ c'\right)$ or $\left(\epsilon^{-\frac{1}{2}} a,\ b + 4\pi v h,\ c - 4\pi v g\right)$

$\left(f_1,\ g_1,\ h_1\right)$ for $\left(\epsilon^{-\frac{1}{2}} f',\ g',\ h'\right)$ or $\left(\epsilon^{-\frac{1}{2}} f,\ g - \frac{v}{4\pi c^2} c,\ h + \frac{v}{4\pi c^2} b\right)$

dt_1 for $\epsilon^{-\frac{1}{2}} dt''$ or $\epsilon^{-\frac{1}{2}} \left(dt' - \frac{v}{c^2} \epsilon dx'\right)$,

where $\epsilon = \left(1 - v^2/c^2\right)^{-1}$; and it will be seen that the factor ε is absorbed, so that the scheme of equations, referred to moving axes, which connects together the new variables with subscripts, is identical in form with the Maxwellian scheme of relations for the aethereal vectors referred to fixed axes. This transformation, from *(x', y', z')* to $\left(x_1,\ y_1,\ z_1\right)$ as dependent variables, signifies an elongation of the space of the problem in the ratio $\epsilon^{\frac{1}{2}}$ along the direction of the motion of the axes of coordinates. Thus if the values of $\left(f_1,\ g_1,\ h_1\right)$ and $\left(a_1,\ b_1,\ c_1\right)$ given as functions of $x_1,\ y_1,\ z_1,\ t_1$ express the course of spontaneous change of the aethereal vectors of a system of moving electrons referred to axes $\left(x_1,\ y_1,\ z_1\right)$ at rest in the aether, then

$$\left(\epsilon^{-\frac{1}{2}} f,\ g - \frac{v}{4\pi c^2} c,\ h + \frac{v}{4\pi c^2} b\right)$$

and

$$\left(\epsilon^{-\frac{1}{2}} a,\ b + 4\pi v h,\ c - 4\pi v g\right),$$

expressed by the same functions of the variables

$$\epsilon^{\frac{1}{2}} x',\ y',\ z',\ \epsilon^{-\frac{1}{2}} t' - \frac{v}{c^2} \epsilon^{\frac{1}{2}} x',$$

will represent the course of change of the aethereal vectors (*f, g, h*) and (*a, b, c*) of a correlated system of moving electrons referred to axes of *(x', y', z')* moving through the aether with uniform translatory velocity (*v*, 0, 0). In this correlation between the courses of change of the two systems, we have

$$\frac{d\left(\epsilon^{-\frac{1}{2}} f\right)}{d\left(\epsilon^{\frac{1}{2}} x'\right)} \text{ equal to } \frac{df_1}{dx_1} - \frac{v}{c^2} \frac{df_1}{dt_1},$$

$$\frac{d}{dy'}\left(g - \frac{v}{4\pi c^2}c\right) \quad ,, \quad \frac{dg_1}{dy_1}$$
$$\frac{d}{dz}\left(h + \frac{v}{4\pi c^2}b\right) \quad ,, \quad \frac{dh_1}{dz_1},$$

where

$$\frac{dc}{dy'} - \frac{db}{dz'} = 4\pi\left(\frac{df}{dt'} - v\frac{df}{dx'}\right)$$

and also

$$\frac{df_1}{dt_1} = \frac{df}{dt};$$

hence $\frac{df}{dx'} + \frac{dg}{dy'} + \frac{dh}{dz'} - \frac{v}{c^2}\left(\frac{df}{dt'} - v\frac{df}{dx'}\right)$ is equal to

$$\epsilon\frac{df_1}{dx_1} + \frac{dg_1}{dy_1} + \frac{dh_1}{dz_1} - \frac{v}{c^2}\epsilon\frac{df}{dt},$$

so that, up to the order of $(v/c)^2$ inclusive,

$$\frac{df}{dx'} + \frac{dg}{dy'} + \frac{dh}{dz'} = \frac{df_1}{dx_1} + \frac{dg_1}{dy_1} + \frac{dh_1}{dz_1}.$$

Thus the conclusions as to the corresponding positions of the electrons of the two systems, which had been previously established up to the first order of *v/c*, are true up to the second order when the dimensions of the moving system are contracted in comparison with the fixed system in the ratio $\epsilon^{-\frac{1}{2}}$ or $1 - \frac{1}{2}v^2/c^2$, along the direction of its motion.

111. The ratio of the strengths of corresponding electrons in the two systems may now be deduced just as it was previously when the discussion was confined to the first order of *v/c*. For the case of a single electron in uniform motion the comparison is with a single electron at rest, near which $(a_1,\ b_1,\ c_1)$ vanishes so far as it depends on that electron: now we have in the general correlation

$$g = g_1 + \frac{v}{4\pi c^2}(c_1 + 4\pi vg),$$

hence in this particular case

$$(g,\ h) = \epsilon(g_1,\ h_1), \text{while } f = \epsilon^{\frac{1}{2}}f_1.$$

But the strength of the electron in the moving system is the value of the integral $\int\int(f\ dy'\ dz' + g\ dz'\ dx' + h\ dx'\ dy')$ extended over any surface closely surrounding its nucleus; that is here $\epsilon^{\frac{1}{2}}\int\int(f_1dy_1dz_1 + g_1dz_1dx_1 + h_1dx_1dy_1)$, so that the strength of each moving electron is $\epsilon^{\frac{1}{2}}$ times that of the correlative fixed electron. As before, no matter what other

electrons are present, this argument still applies if the surface be taken to surround the electron under consideration very closely, because then the wholly preponderating part of each vector is that which belongs to the adjacent electron.[4]

112. We require however to construct a correlative system devoid of the translatory motion in which the strengths of the electrons shall be equal instead of proportional, since motion of a material system containing electrons cannot alter their strengths. The principle of dynamical similarity will effect this.

We have in fact to reduce the scale of the electric charges, and therefore of $\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}$, in a system at rest in the ratio $\epsilon^{-\frac{1}{2}}$. Apply therefore a transformation

$$(x,\ y,\ z)=k\left(x_1,\ y_1,\ z_1\right),\ t=lt_1,$$

$$(a,b,c)=\vartheta(a_1,\ b_1,\ c_1),\ (f,\ g,\ h)=\epsilon^{-\frac{1}{2}}k\left(f_1,\ g_1,\ h_1\right);$$

and the form of the fundamental circuital aethereal relations will not be changed provided $k=l$ and $\vartheta=\epsilon^{-\frac{1}{2}}k$. Thus we may have *k* and *l* both unity and $\vartheta=\epsilon^{-\frac{1}{2}}$; so that no further change of scale in space and time is required, but only a diminution of (*a, b, c*) in the ratio $\epsilon^{-\frac{1}{2}}$.

We derive the result, correct to the second order, that if the internal forces of a material system arise wholly from electrodynamic actions between the systems of electrons which constitute the atoms, then an effect of imparting to a steady material system a uniform velocity of translation is to produce a uniform contraction of the system in the direction of the motion, of amount $\epsilon^{-\frac{1}{2}}$ or $1-\frac{1}{2}v^2/c^2$. The electrons will occupy corresponding positions in this contracted system, but the aethereal displacements in the space around them will not correspond: if (*f, g, h*) and (*a, b, c*) are those of the moving system, then the electric and magnetic displacements at corresponding points of the fixed systems will be the values that the vectors

$$\epsilon^{\frac{1}{2}}\left(\epsilon^{-\frac{1}{2}}f,\ g-\frac{v}{4\pi c^2}c,\ h+\frac{v}{4\pi c^2}b\right)$$

and

$$\epsilon^{\frac{1}{2}}\left(\epsilon^{-\frac{1}{2}}a,\ b+4\pi vh,\ c-4\pi vg\right)$$

had at a time const. $+vx/c^2$ before the instant considered when the scale of time is enlarged in the ratio $\epsilon^{\frac{1}{2}}$.

As both the electric and magnetic vectors of radiation lie in the wave-front, it follows that in the two correlated systems, fixed and moving, the relative wave-fronts of radiation correspond, as also do the rays which are the paths of the radiant energy relative to the systems. The change of the time variable, in the comparison of radiations in the fixed and moving systems, involves the Doppler effect on the wave-length.

The Correlation between a stationary and a moving Medium, as regards trains of Radiation

113. Consider the aethereal displacement given by

$$(f_1,\ g_1,\ h_1)=(L,\ M,\ N)\ F\ (lx_1+my_1+nz_1-pt),$$

which belongs to a plane wave-train advancing, along the direction (*l, m, n*) with velocity *V*, or *c/μ* where *μ* is refractive index, equal to

$$p(l^2+m^2+n^2)^{-\frac{1}{2}},$$

in the material medium at rest referred to coordinates $(x_1,\ y_1,\ z_1)$. In the corresponding wave-train relative to the same medium in motion specified by coordinates *(x, y, z)*, and considered as shrunk in the above manner as a result of the motion, the vectors (*f, g, h*) and (*a, b, c*) satisfy the relation

$$\epsilon^{\frac{1}{2}}\left(\epsilon^{-\frac{1}{2}}f,\ g-\frac{v}{4\pi c^2}c,\ h+\frac{v}{4\pi c^2}b\right)$$
$$=(L,\ M,\ N)\ F\left\{l\epsilon^{\frac{1}{2}}x+my+nz-p\epsilon^{-\frac{1}{2}}\left(t-\frac{v}{c^2}\epsilon x\right)\right\}$$
$$=(L,\ M,\ N)\ F\left\{\left(l\epsilon^{\frac{1}{2}}+\frac{pv}{c^2}\epsilon^{\frac{1}{2}}\right)x+my+nz-p\epsilon^{-\frac{1}{2}}t\right\}$$.}}

As the wave-train in the medium at rest is one of transverse displacement, so that the vectors $(f_1,\ g_1,\ h_1)$ and $(a_1,\ b_1,\ c_1)$ are both in the wave-front, the same is therefore true for the vectors (*f, g, h*) and (*a, b, c*) in the correlative wave-train in the moving system, as was in fact to be anticipated from the circuital quality of these vectors: the direction vector of the front of the latter train is proportional to $\left(l\epsilon^{\frac{1}{2}}+\frac{pv}{c^2}\epsilon^{\frac{1}{2}},\ m,\ n\right)$, and its velocity of propagation is

$$p\epsilon^{-\frac{1}{2}}/\left\{\left(l\epsilon^{\frac{1}{2}}+\frac{pv}{c^2}\epsilon^{\frac{1}{2}}\right)^2+m^2+n^2\right\}^{\frac{1}{2}}.$$

Thus, when the wave-train is travelling with velocity *V* along the direction of translation of the material medium, that is along the axis of *x* so that *m* and *n* are null, the velocity of the train relative to the moving medium is

$$V\epsilon^{-1}/\left(1+\frac{Vv}{c^2}\right),$$

which is, to the second order,

$$V\left(1+\frac{v^2}{c^2}\right)/\left(1+\frac{Vv}{c^2}\right) \text{or } V-\frac{v}{\mu^2}-\left(\frac{1}{\mu}-\frac{1}{\mu^3}\right)\frac{v^2}{c}.$$

The second term in this expression is the Fresnel effect, and the remaining term is its second order correction on our hypothesis which includes Michelson's negative result.

In the general correlation, the wave-length in the train of radiation relative to the moving material system differs from that in the corresponding train in the same system at rest by the factor

$$\left(1+2l\frac{pv}{c^2}\right)^{-\frac{1}{2}}, \text{ or } 1-lv/\mu c,$$

where l is the cosine of the inclination of the ray to the direction of v; it is thus shorter by a quantity of the first order, which represents the Doppler effect on wave-length because the period is the same up to that order.

When the wave-fronts relative to the moving medium are travelling in a direction making an angle θ', in the plane xy so that n is null, with the direction of motion of the medium, the velocity V' of the wave-train (of wave-length thus altered) relative to the medium is given by

$$\frac{\cos\theta'}{V'} = \frac{l\epsilon}{p} + \frac{v\epsilon}{c^2},\ \ \frac{\sin\theta'}{V'} = \frac{m\epsilon^{\frac{1}{2}}}{p},$$

where $\left(l^2+m^2\right)p^2 = V^{-2}$. Thus

$$\left(\frac{\epsilon^{-1}\cos\theta'}{V'} - \frac{v}{c^2}\right)^2 + \frac{\epsilon^{-1}\cos^2\theta'}{V'^2} = \frac{1}{V^2},$$

so that neglecting $(v/c)^3$,

$$V' = V - \frac{v}{\mu^2}\cos\theta' - \frac{1}{2}\left(1-\mu^{-2}\right)\frac{v^2}{\mu c}\left(1+3\cos^2\theta'\right),$$

where $\mu = c/V$, of which the last term is the general form of the second order correction to Fresnel's expression. In free aether, for which μ, is unity, this formula represents the velocity relative to the moving axes of an unaltered wave-train, as it ought to do.

As (f, g, h) and (a, b, c) are in the same phase in the free transparent aether, when one of them is null so is the other: hence in any experimental arrangement, regions where there is no disturbance in the one system correspond to regions where there is no disturbance in the other. As optical measurements are usually made by the null method of adjusting the apparatus so that the disturbance vanishes, this result carries the general absence of effect of the Earth's motion in optical experiments, up to the second order of small quantities.

Influence of translatory motion on the Structure of a Molecule: the law of Conservation of Mass

114. As a simple illustration of the general molecular theory, let us consider the group formed of a pair of electrons of opposite signs describing steady circular orbits round each other in a position of rest:[5] we can assert from the correlation, that when this pair is moving through the aether with velocity v in a direction lying in the plane of their orbits, these orbits relative to the translatory motion will be flattened along the direction of v to ellipticity $1-\frac{1}{2}v^2/c^2$, while there will be a first-order retardation of phase in each orbital motion when the electron is in front of the mean position combined with acceleration when behind it so that on the whole the period will be changed only in the second-order ratio $1+\frac{1}{2}v^2/c^2$. The specification of the orbital modification produced by the translatory motion, for the general case when the direction of that motion is inclined to the plane of the orbit, may be made similarly: it can also be extended to an ideal molecule constituted of any orbital system of electrons however complex. But this statement implies that the nucleus of the electron is merely a singular point in the aether, that there is nothing involved in it of the nature of inertia foreign to the aether: it also implies that there are no forces between the electrons other than those that exist through the mediation of the aether as here defined, that is other than electric forces.

The circumstance that the changes of their free periods, arising from convection of the molecules through the aether, are of the second order in *v/c*, is of course vital for the theory of the spectroscopic measurement of celestial velocities in the line of sight. That conclusion would however still hold good if we imagined the molecule to have inertia and potential energy extraneous to (i.e. unconnected with) the aether of optical and electrical phenomena, *provided these properties are not affected by the uniform motion*: for the aethereal fields of the moving electric charges, free or constrained, existing in the molecule, will be symmetrical fore and aft and unaltered to the first order by the motion, and therefore a change of sign of the velocity of translation will not affect them, so that the periods of free vibration cannot involve the first power of this velocity.

115. The fact that uniform motion of the molecule through the aether does not disturb its constitution to the first order, nor the aethereal symmetry of the moving system fore and aft, shows that when steady motion is established the mean kinetic energy of the system consists of the internal energy of the molecule, which is the same as when it is at rest, together with the sum of the energies belonging to the motions of translation of its separate electrons. This is verified on reflecting that the disturbance in the aether is made up additively of those due to the internal motions of the electrons in the molecule and those due to their common velocity of translation. Thus in estimating the mean value of the volume-integral of the square of the aethereal disturbance, which is the total kinetic energy, we shall have the integrated square of each of these disturbances separately, together with the integral of terms involving their product. Now one factor of this product is constant in time and symmetrical fore and aft as regards each electron, that factor namely which arises from the uniform translation; the other factor, arising from the orbital motions of the electrons, is oscillatory and symmetrical in front and rear of each orbit: thus the integrated product is by symmetry null. This establishes the result stated, that the kinetic energy of the moving molecule is made up of an internal energy, the same up to the first order of the ratio of its velocity to that of radiation as if it were at rest, and the energy of translation of its electrons. The coefficient of half the square of the velocity of translation in the latter part is therefore, up to that order, the measure of the inertia, or mass, of the molecule thus constituted. Hence when the square of the ratio of the velocity of translation of the molecule to that of radiation is neglected, its electric inertia is equal to the sum of those of the electrons which compose it; and the fundamental chemical law of the constancy of mass throughout molecular transformations is verified for that part of the mass (whether it be all of it or not) that is of electric origin.

116. Objection has been taken to the view that the whole of the inertia of a molecule is associated with electric action, on the ground that gravitation, which has presumably no relations with such action, is proportional to mass: it has been suggested that inertia and gravity may be different results of the same cause. Now the inertia is by definition the coefficient of half the square of the velocity in the expression for the translatory energy of the molecule: in the constitution of the molecule it is admitted, from electrolytic considerations, that electric forces or agencies prevail enormously over gravitative ones: it seems fair to conclude that of its energy the electric part prevails equally over the gravitative part: but this is simply asserting that inertia is mainly of electric, or rather of aethereal, origin. Moreover the increase of kinetic electric energy of an electron arising from its motion with velocity *v* depends on v^2/c^2, on the coefficient of inertia of the aether, and on the dimensions of its nucleus, where *c*is the velocity of radiation: the increase of its gravitational energy would presumably in like manner depend on v^2/c'^2, where *c′* is the velocity of propagation of gravitation and is enormously greater than *c*. On neither ground does it appear likely that mass is to any considerable degree an attribute of gravitation.

The Transition from Electrons to Molecules

117. The main additional result derived from this second-order discussion is that if we assume all molecular forces to be electric forces, motion of a material system through the aether alters its dimensions in a minute but definite manner. A scrutiny, on all sides, of the basis of this inference is of course desirable. As a preliminary it is to be noticed that the molecular forces on the action of which it depends are extremely great in comparison with any distributions of force arising from finite currents or electrifications produced in the system as a whole. In the comparison between the two identical systems, one at rest the other in motion, of the analogy above developed, their electrons occupy corresponding positions in their spaces at all times: thus at first sight it is only systems in which the electrons are absolutely at rest that can be thus compared. But even in the case of dielectric bodies at rest, though the

molecules are fixed the electrons are revolving in the molecules: yet that does not sensibly affect the application of the correspondence. For the only difference thereby introduced in it is that the phases of the orbital motions of those molecules of the moving material system that are situated further in advance, in the direction of the movement of the system, are slightly accelerated in comparison with the corresponding phases in the fixed system. Now the permanent or secular relations between molecules, supposed far enough apart not to interfere in a structural manner with each other so as to form compound molecules, are independent of these relative phases: to obtain them we in fact replace each molecule by its steady secular equivalent in the Gaussian sense, as has to be done in a representation of their magnetism, and thus the phase-change makes no difference for the present purpose. The case is however different when there are electric currents flowing in the system, for that involves the transfer of some electrons into entirely new positions, it may be at a finite distance: these wandering electrons or ions interfere with the exact statement of the correlation, and they interfere to a like extent with any conclusions that may be drawn from it, as to change of form of solid bodies carrying currents arising from their motion with the Earth through the aether.

How far then is the correlation between the fixed material system and the moving system modified by electric conduction? In the theorem the position of each electron in the material medium in motion, at time t, corresponds with that which it would occupy in the medium at rest at time $t - vx/c^2$. When the material medium is a solid dielectric mass, the mean position of the electron is the same at all times, and as we have seen this element of time does not enter into the comparison at all: but when the medium is conducting, the electric currents in it involve migration of electrons through it, and we must consider how far the correspondence is thereby prejudiced. Only two views of the nature of conduction, in this connexion, are open. The current in metals may possibly (but not likely) be carried by very few electrons, in which case they will migrate with sensible speed; but the smallness of their number, compared with the total number of combined electrons, prevents their changes of position from sensibly affecting the molecular structure of the medium: we know in fact that the mechanical structure of a conductor is not sensibly affected when it carries a current. On the other hand a considerable proportion of the electrons may take part in carrying the current; in which case their velocity of migration is excessively minute, as for instance follows from the phenomena of migration in electrolysis;[6] and the discrepancy of position of those electrons, in the application of the correlation theorem, involving the factor v/c^2 as well as this velocity, is negligible to an order higher than the second, just as was the discrepancy of phase in the individual molecular orbits. To reach this conclusion, it is by no means necessary to assume that we have any knowledge of the process by which ionisation, or the passing on of electrons from molecule to molecule, occurs in conductive processes.

Influence of Convection on Conductivity

118. In this connexion we can gain some knowledge of the nature and amount of the effect of the Earth's motion on electrolytic conduction. If the convective velocity v is in the direction of the current, and the actions between the ions are, as usual in electrolytic theory, assumed to be wholly electric, and w and w' represent velocities of positive and negative ions, then the position of the positive ion in the electrolyte at rest is given by $x = wt$; hence (§ 112) in the electrolyte in motion with the same electric force it is given by $x = w\left(1 - \frac{v}{c^2}x\right)$, so that $x = \frac{w}{1 + vw/c^2}t$; thus the velocity of the positive ion relative to the moving electrolyte is $w/\left(1 + \frac{vw}{c^2}\right)$. The velocity of the negative ion is similarly $w'/\left(1 - \frac{vw'}{c^2}\right)$. The electric current, being determined by the sum of these velocities, is altered as regards these ions in the ratio of $w + w' - \frac{v}{c^2}\left(w^2 - w'^2\right)$ to $w + w'$ approximately; it is thus

diminished in the ratio $1 - v(w - w')/c^2$; and the conductivity of the electrolyte is diminished in this ratio, where now $w - w'$ represents an average value, the difference of the velocities of drift of positive and negative ions. This change of conductivity is a unilateral one, being reversed when the direction of the current is reversed: it is at most of the second order of small quantities: it vanishes altogether, or rather becomes of two orders higher, when the velocities of the positive and negative ions are the same. It may be remarked incidentally that, as the numbers of positive and negative ions taking part in the current of conduction are the same, the specification of that current with reference to moving matter is just the same as with reference to the stationary aether.

The Argument of Lorentz regarding the Michelson experiment

119. As an assistance to the formation of a judgment on these questions, it will be convenient to insert here a free translation of the considerations by which Lorentz[7] supported the possibility of an explanation, of the kind above developed, of the negative result of Michelson's experiments on the influence of material convection on phenomena of optical interference.

"However extraordinary this hypothesis may appear at first sight, it must be admitted that it is by no means gratuitous, if we assume that the intermolecular forces act through the mediation of the aether in a manner similar to that which we know to be the case in regard to electric and magnetic forces. If that is so, the translation of the matter will most likely alter the action between two molecules or atoms in a manner similar to that in which it alters the attraction or repulsion between electrically charged particles. As then the form and the dimensions of a solid body are determined in the last resort by the intensity of the molecular forces, an alteration of the dimensions cannot well be left out of consideration.

"In its theoretical aspect there is thus nothing to be urged against the hypothesis. As regards its experimental aspect we at once notice that the elongation or contraction which it implies is extraordinarily minute. It would involve a shortening in the diameter of the Earth of about $6\frac{1}{2}$ centimetres. The only experimental arrangements in which it could come into evidence would be just of the type of this one of Michelson's which first suggested it.

"It is worthy of remark, that we are led precisely to this law of alteration of dimensions when we assume *first* that, without taking account of molecular motions, in a solid body left to itself the forces of attraction and repulsion acting on each molecule maintain themselves in equilibrium, and *secondly* — for which there is admittedly no evidence — that the same law applies to these molecular forces, as regards their alteration by convection, that has been demonstrated for the electrostatic attractions of moving charges. Let us understand by S_1, and S_2, not as previously two systems of charged particles, but two systems of molecules, — the second at rest and the first in motion with velocity *v* in the direction of the axes of *x*, — between whose dimensions the previously given relation holds; then since in both systems the *x* components of the forces are the same, while the *y* and *x* components differ by the factors given, it is clear that the forces in S_1 will balance when that is the case for those of S_2. If therefore S_2 is the state of equilibrium of a solid body at rest, the molecules in *S_1* have just those positions in which they could subsist under the influence of the motion of translation. The displacement into this new configuration would therefore take place of itself, involving a contraction in the direction of motion in the ratio of unity to $\left(1 - v^2/c^2\right)^{\frac{1}{2}}$.

"In reality the molecules of a body are not at rest, but corresponding to each position of equilibrium they are in a state of stationary motion. How far this difference is of importance for the phenomena treated, must be left undetermined: the experiments of Michelson and Morley leave for it a comparatively wide range of effect on account of the unavoidable errors of observation."

The force of the last remark is removed by Michelson's more recent observations[8] with a longer ray-path, in which the delicacy was so great that it was necessary for consistent results to get rid of the air; even then no trace of uncompensated effect was observed.

Are the linear equations of the Aether exact?

120. In favour of the view that the interactions between atoms are in very great part those necessitated by the aether whose properties are revealed in electric and optical phenomena, there is, in addition to the inherent theoretical difficulty in conceiving any other kind of interaction, the actual fact that on the lines of the above argument such a view does account for a definite and well-ascertained experimental result, that of Michelson, above discussed, which has hitherto stood by itself as the only quantitative observational evidence that has a bearing on this question. It can be said on the other side that this view of aethereal action does not directly cover gravitational phenomena, unless the rather artificial pulsatory theory of gravity is allowed.[9] But there is another aspect of the matter. The equations of the free aether, as revealed by MacCullagh's optical analysis, are linear equations: they in fact must be so if all kinds of radiations are to travel with the same speed in the celestial spaces. In Maxwell's hands, equivalent relations with the appropriate generalization were arrived at on the electric side, and formed a basis for the explanation of the whole *plexus* of electrodynamic and optical phenomena. Further theoretical discussion has in all directions tended to widen the scope and enhance the inherent simplicity of this scheme. The question arises whether there is anything to gainsay a view that this simple linear scheme is only the first approximation, a very close one however, to an analytical specification of the aether: just as the linear scheme of equations of the theory of propagation of sound covers the whole of the phenomena of acoustics, although in arriving at those equations from the dynamics of the atmosphere all terms involving the square of the ratio of the velocity of the actual aereal disturbance to the velocity of its propagation are neglected, for the reason that their consequences are outside the limits of observation in that domain. Why then should not relatively minute phenomena like gravitation be involved in similar non-linear terms, or terms involving differentials of higher orders, in the analytical specification of the free aether, which are as insignificant compared with the main fully ascertained linear terms as is the gravitation between two electric systems compared with their mutual electric forces? Against this there is a subjective reluctance to disturb the ideal simplicity of the aethereal scheme: but there is no help for that if its content is not sufficiently extensive for the facts. Of more weight is the circumstance that a train of radiation from a distant star would change its form as it advanced across space, that there would in fact be optical dispersion in the free aether if such second-order terms existed. The amount of such dispersion that would be at all allowable is known to be excessively minute, from the circumstance that celestial bodies on emerging from eclipse or occupation show no changes of colour: the smallness of the amount that would be required may be estimated by comparing the electric force between two ions with their gravitational attraction. Unless the effects of such terms of higher order, in the equations of aethereal activity, increased enormously in importance at molecular distances, relatively to the main linear terms, the proposition that the interactions of molecules are mainly of electric quality would remain valid: now such increase of importance does not seem likely as regards the mechanism of gravitation, for gravity and electric force both obey the same law of the inverse square of the distance, a law which in fact belongs, of mathematical necessity, to the steady permanent interactions between any kinds of molecular nuclei of elastic or motional disturbance in an extended medium, which are of the type of simple poles.

A question of some interest arises, as to whether the assumption that the linear equations of free aether are a first approximation, obtained by the omission of non-linear terms, would imply a virtual recognition of structure in that medium. A presumption of this kind would be useless except for purposes of vivid illustration after the manner of mechanical models, so long as there is absolutely no means of experimenting on the properties of free aether: and this practically comes to the same thing as taking such structure to be non-existent.

121. There is thus little to be urged in favour of leaving this loophole for the explanation of gravitation. On the other side moreover there appears to be the fatal objection that any action accounted for in this way would have relations with radiation, including a velocity of transmission of the same order as that of light. The knowledge that the speed of transmission of gravitation, if finite at all, enormously transcends that of radiation, shows that it forms no objection to a theory of electric and radiant phenomena that gravitation is

not found to be involved in it. An analogy in fact suggests itself with the molecular electric theory as developed by Weber, Kirchhoff and their school, which gave a complete account of ordinary material electric phenomena, and only failed when the totally different region of radiation came into the discussion. It seems fair to conclude, in the one case as in the other, that in the constitution of the energy-relations on which the phenomena depend, a new property of the medium becomes explicitly involved in the more refined theory (not merely implicitly as in the energy-function that suffices for ordinary material electrodynamics) such for instance as the incompressibility that is utilized in the pulsatory theory or illustration of gravitation. The general reasons against the notion that the fundamental property of mass in matter is in direct connexion with the mechanism by which gravitation is transmitted have been given above (§ 116). There appears then, as yet, to be nothing to tempt us to depart from the natural prepossession, by considering the simple linear equations of the aether to be other than exact.

Dimensional Relations: in connexion with the definite scale of magnitude of Atomic Structure

122. Important considerations bearing on the question as to how far atoms of matter are constituted simply of singularities in the aether, practically point-nuclei, may be derived from the Newtonian principle of dynamical similarity, as utilized above (§ 112). Let us compare two such aethereal systems represented one by ordinary the other by subscripted variables, between which there is a correspondence given by

$$(x,\ y,\ z) = k\left(x_1,\ y_1,\ z_1\right),\ t = lt_1,$$
$$(a,\ b,\ c) = \vartheta\left(a_1,\ b_1,\ c_1\right),\ (f,\ g,\ h) = \phi\left(f_1,\ g_1,\ h_1\right);$$

the aethereal equations for the one system will be identical with the aethereal equations for the other provided

$$\vartheta/k = \phi/l,\ \phi/k = \vartheta/l,$$

so that

$$\vartheta = \phi \text{ and } k = l.$$

Hence, given any one existing system of electrons with pointnuclei, another system is possible in the same aether having all distances and times reduced in any the same ratio, and electric displacement and magnetic flux independently reduced in any other the same ratio. But if the electrons of this correlated system are to be of the same strengths as the original ones ϕ/k must be unity; hence the scale must be altered in the same ratio throughout, as regards length, time, and the inductions. Thus, given any existing steady system of electrons, the same system altered to any other scale of linear magnitude is possible if there are none but electric actions. This is on the hypothesis which is here generally adopted, that the dimensions of the nucleus of an electron are so small, compared with the mutual distances of electrons, that these dimensions are not sensibly involved in the forces between them. If this condition is left out the constancy of volume of the nucleus will have to be taken into consideration in the dimensional transformation, so that *k* must be unity; and this indefiniteness of linear scale in a material body cannot exist. The size of a molecule would also be rendered determinate if residual non-linear terms in the aethereal equations became sensible at intermolecular distances. Thus, these saving hypotheses being excluded, if the atoms of matter were constituted electrically, and the forces between them were wholly of electric origin, there would be nothing to determine the scale of an isolated system as regards time and space: and different systems need not be always of the same scale of magnitude as regards their atomic structure.

123. A similar deficiency of definite scale would also be expected to exist in any hydrodynamical theory or illustration which would construct an atom out of vortex rings. Thus let us consider a system of vortex rings, (ξ, η, ζ) being the vorticity at the point (x, y, z), and compare with another system in another space (x', y', z') such that the coordinates of corresponding points are connected by the relation

$$(x',\ y',\ z') = k(x,\ y,\ z),$$

while the vorticities at these points are connected by the relation

$$(\xi',\ \eta', \zeta') = \kappa(\xi,\ \eta,\ \zeta),$$

The formula of von Helmholtz for the velocity (u, v, w) of the fluid in terms of the vorticity, being of type

$$u = \int \left(\eta \frac{d}{dz} - \zeta \frac{d}{dy}\right) \frac{1}{r} d\tau,$$

gives

$$(u',\ v',\ w') = k\kappa(u,\ v,\ w).$$

But the systems will maintain their correspondence of configuration throughout succeeding time, only provided always

$$(u',\ v',\ w') = k(u,\ v,\ w);$$

hence $\kappa = 1$ while k is arbitrary. Thus if any vortex-system is compared with another one expanded as regards linear scale k times, and the vorticity is at each point unaltered, so that the circulations of the vortices in the new system are all increased k^2 times, then their subsequent histories will correspond exactly.

The circulation of the vortex is however in the dynamical theory an unalterable constant, so that the one system cannot be changed by natural processes into the other. Let us try therefore to avoid this difference by a change of the time scale as well, so that $t' = \lambda t$; then for continued correspondence

$$(u',\ v',\ w') = k\lambda^{-1}(u,\ v,\ w):$$

hence $k\kappa = k\lambda^{-1}$ so that $\kappa = \lambda^{-1}$; and the strengths of the vortices are altered in the ratio $k^2\lambda^{-1}$, which must be a constant. Thus if the scale of time is increased λ times, and that of linear magnitude $\lambda^{-\frac{1}{2}}$ times, and the corresponding vortex filaments are of the same strengths, the systems will continue permanently in correspondence. This is however on the assumption that the vorticity is around a vacuous core, or a fluid core so thin that its actual section does not affect the mutual actions of the vortices: for the change of linear scale will alter the volume of the core of each ring. There is under these conditions nothing in the hydrodynamical forces to fix the scale of magnitude of an isolated vortex-system with vacuous cores; the same system can equally exist with linear dimensions k times as great when all the time constants will be diminished k^2 times.

124. The definiteness of scale of the molecules of material systems thus precludes the possibility of their being constituted of singularities of a uniform continuum, of either of these kinds with nuclei undistinguishable from mathematical points. The constancy of inertia and gravity throughout all chemical transformations forms practically sufficient evidence for the physicist that all matter is built up out of the same primordial stuff: this stuff, if it is constituted of intrinsic singularities in a uniform aethereal continuum with relations exactly linear, must thus be made up of elements of type rather more complex than simple positional and motional singularities with nuclei devoid of sensible volume. Another element apart from finiteness of dimensions of nuclear structure that could enter, on the theory of an aether exactly linear in its relations, is that of time: for example it has been seen how the gravitation of atoms can be

imitated by supposing a definite periodic time of pulsation to be associated with each electron. A change of scale such as that above discussed would then change the forces of gravitation, unless possibly the time of pulsation could be suitably altered and the change thus counteracted.

125. In the above considerations there is strong evidence that gravitation is not to be expected to be appreciably involved within the scheme which suffices to cover the phenomena of electrodynamics and optics. The introduction of the timerelation inherent in pulsating nuclei seems still to be the only obvious way of representing it, in default of its arising from second-order terms in the dynamical relations of the aether. The permanence of scale of magnitude of the material atoms of various types involves the presence of actions depending on the magnitude and structure of the electric nuclei, which though they may be purely aethereal are local, and thus not pertinent to general electrical and optical theory: the existence of a configuration of minimum energy in the molecule in fact implies finite structure in the nuclei in some such way. At the same time the Michelson interference-result indicates that these other agencies play a quite subordinate part in our present problems: for the correlation above established, which involves that result, only holds strictly for electrons whose nuclei are considered as mere points in comparison with their mutual distances. Atomic inertia other than that which comes from the aether in some way it seems impossible to conceive: but in other respects we are hardly on the threshold of the structure of the atom. The problem there involved is not to assign a structure so minutely definite that it will include the whole complex of chemical actions, but rather to ascertain how much must be postulated in order to correlate the main features of those universal agencies, affecting all kinds of matter, with which the theoretical side of physical science deals.

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1. Namely as the distance *r* from it diminishes indefinitely, the magnetic induction tends to the form $evr^{-2}\ sin\theta$, at right angles to the plane of the angle θ between *r* and the velocity *v* of the electron: this arises as the disturbance of the medium involved in annulling the electron in its original position and restoring it in the new position to which it has moved. The relations will appear more clearly when visualized by the kinematic representation of Appendix E; or when we pass to the limit in the formulae of Chapter ix relating to the field of a moving charged body of finite dimensions.

   The specification in the text, as a simple pole, only applies for an electron moving with velocity *v*, when terms of the order $(v/c)^2$ are neglected: otherwise the aethereal field close around it is not isotropic and an amended specification derivable from the formulae of Chapter ix must be substituted. In the second-order discussion of Chapter xi this more exact form is implicitly involved, the strength of the electron being determined (§ 111) by the concentration of the aethereal displacement around it. The singularity in the magnetic field which is involved in the motion of the electron, not of course an intrinsic one, has no concentration.
2. See Appendix E.
3. Cf. footnote, p. 162.
4. This result follows more immediately from § 110, which shows that corresponding densities of electrification are equal, while corresponding volumes are as $\epsilon^{\frac{1}{2}}$ to unity.
5. The orbital velocities are in this illustration supposed so small that radiation is not important. Cf. §§ 151 — 6 *infra*.
6. Cf. Appendix B, § 6.
7. 'Versuch einer Theorie...' 1895, §§ 91—2.
8. American Journal of Science, 1897.
9. Cr. *Phil. Trans*. 1897 A, p. 317.